# LEPTON- AND FAMILY-NUMBER VIOLATION FROM EXOTIC SCALARS

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Received 27 June 1985

We analyse all possibilities of lepton and family non-conservation in the standard SU(3)×SU(2)×U(1) gauge model induced by the existence of new scalars which can couple to the standard leptons, with emphasis on neutrino masses, muon decays ($\mu^- \to e^- \gamma$, $\mu^- \to e^- e^- e^+$) and muonium–anti-muonium oscillations.

In the standard SU(3) × SU(2) × U(1) model of electroweak and strong interactions [1], lepton number is automatically conserved. In addition, the family numbers are separately conserved. Although it appears rather difficult to observe lepton- or family-number violating processes in the immediate future, we should not lose sight of the fact that there is no satisfactory reason for the existence of these global symmetries and they may very well be a low energy phenomenon. Of course, explicit violation [2] of lepton- as well as baryon-number is one of the basic features of GUT's. In this paper we confine ourselves to the standard SU(3) × SU(2) × U(1) gauge group with three families of leptons and explore all possibilities that can lead to lepton- or family-number non-conservation in view of the present available experimental limits [3].

The leptonic sector of the standard model consists of

$$L = \begin{pmatrix} \nu_L^0 \\ e_L^0 \end{pmatrix} = (2, -\tfrac{1}{2}; 0),$$

$$e_R^0 = (1, -1; +1),$$

where the index zero specifies that these are the weak eigenstates. The last entry refers to lepton number, coupled to possibly more than one Higgs doublet

$$H \equiv \begin{pmatrix} H_+^{(k)} \\ H_0^{(k)} \end{pmatrix} = (2, +\tfrac{1}{2}; 0)$$

via the Yukawa lagrangian

$$\mathcal{L}_0 = (\bar{\nu}_i^0, \bar{e}_i^0)_L \left\{ \sum_k \alpha_{ij}^{(k)} \begin{pmatrix} H_+^{(k)} \\ H_0^{(k)} \end{pmatrix} \right\} e_{jR}^0 . \tag{1}$$

The indices $i, j = 1, 2, 3$ are family indices while $k$ numbers the Higgs isodoublets.

Denoting by $U$ the unitary matrix that diagonalizes the charged-lepton mass matrix

$$m_{ij}(e) \equiv \alpha_{ij}^{(1)} v_1 + \alpha_{ij}^{(2)} v_2 + \ldots ,$$

the Yukawa lagrangian in terms of the rotated fields

$$L' \equiv U^* L, \quad e_L' = U e_L \tag{2}$$

becomes

$$\mathcal{L}_0 = (\bar{e}_L)_i m_i^{(D)}(e)(e_R)_i$$
$$- (\nu_L)_i (\alpha_{ij}'^{(1)} H_+^{(1)} + \alpha_{ij}'^{(2)} H_+^{(2)} + \ldots)(e_L)_j + \text{h.c.}, \tag{3}$$

where $e_{iL}$, $e_{iR}$ are the left- and right-handed eigenvectors of $m_{ij}(e)$

$$m^D(e) \equiv U(\alpha^{(1)} H_0^{(1)} + \alpha^{(2)} H_0^{(2)} + \ldots) U^\dagger , \tag{4}$$

and

$$\alpha'^{(1)} \equiv U \alpha^{(1)} U^\dagger , \quad \alpha'^{(2)} \equiv U \alpha^{(2)} U^\dagger , \ldots . \tag{5}$$

The rotated Yukawa matrices $\alpha'^{(1)}, \alpha'^{(2)}, \ldots$ are not in general diagonal and family mixing is present unless the bare Yukawa matrices are proportional to each other or, of course, if one has only one Higgs field.

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The case of flavour mixing Higgs couplings does not appear to be very attractive however, since it offers no explanation of the smallness of the off-diagonal elements in $\alpha'^{(1)}, \alpha'^{(2)}, \ldots$.

Family- and lepton-number violation is, of course, possible if one is willing to extend the field content of the theory by the introduction of exotic fields different from the above mentioned. This can be done by introducing new fermions or new spin-zero bosons. The only interesting fermionic possibility is the well-known case of the "right-handed neutrino" a gauge singlet fermion $N_R \equiv (1, 0; 1)$ in three family varieties [4]. No other new fermion can be introduced with renormalizable and gauge invariant couplings to L and $e_R$. The possibility of additional families reiterates the same question. With the right-handed neutrino, the following terms become possible:

$$\delta\mathcal{L} = (\bar{\nu}_i^0\, \bar{e}_i^0)_L (b_{ij}^{(1)}\bar{H}^{(1)} + b_{ij}^{(2)}\bar{H}^{(2)} + \ldots) N_R^0 + \tfrac{1}{2} M_{ij} N_{iL}^{\overline{0c}} N_{jR}^0 + \text{h.c.} \tag{6}$$

The Higgs doublets $\bar{H} = (2, -\frac{1}{2}; 0)$ could be just $i\tau_2 H^*$ or new fields. The Majorana mass $M_{ij}$ which breaks lepton number explicitly could arise from the expectation value of a gauge-singlet spin-zero field [5] (the majoron) but since no compelling reason in favour of spontaneously broken lepton number exists, we shall try to be as general as possible and consider it as an arbitrary matrix in family space.

If $\Omega$ is the orthogonal matrix that diagonalizes $M$, then the Yukawa lagrangian becomes

$$\delta\mathcal{L} = (\bar{\nu}_i \bar{e}_i)_L (b_{ij}'^{(1)}\bar{H}^{(1)} + b_{ij}'^{(2)}\bar{H}^{(2)} + \ldots) N_{jR} + \tfrac{1}{2} M_i^{(N)} \bar{N}_i N_i + \text{h.c.}, \tag{7}$$

where

$$N_j = N_{jL} + N_{jR}.$$

In terms of the rotated fields $N_R \equiv \Omega N_R^0$, where

$$M^{(N)} \equiv OMO^\dagger \tag{8}$$

and

$$b'^{(1)} = U b^{(1)} O^\dagger, \quad b'^{(2)} = U b^{(2)} O^\dagger, \ldots . \tag{9}$$

The neutrino Dirac mass matrix

$$m_{ij}(v) \equiv b_{ij}'^{(1)} v_1 + b_{ij}'^{(2)} v_2 + \ldots \tag{10}$$

is not in general diagonal, not even in the minimal case of one Higgs doublet, due to the fact that the matrices $U$ and $\Omega$ are independent. Restricting ourselves in the minimal case of one Higgs doublet the resulting Yukawa lagrangian becomes

$$\mathcal{L}_Y = L_i \alpha_i^D H e_R + L_i b_{ij} \bar{H} N_R + \tfrac{1}{2} M_i^{(N)} \bar{N}_i N_i + \text{h.c.} \tag{11}$$

The induced family mixing for the standard leptons is

$$(\bar{L}_i \bar{H})\, b_{ik} (M^{-1})_{kl} b_{lj} H^\dagger L_j^c ,$$

while the neutrino mass eigenvalues are approximately $M$ and $bM^{-1}b\langle\bar{H}_0\rangle^2$. The scale $M$ is determined by the limits on the left-handed neutrino mass.

Lepton- and family-number violation could also arise due to an extended bosonic sector. The only new spin-zero bosons that can couple in a renormalizable and gauge invariant way to the standard leptons are the fields

$$S^- \equiv (1, 1), \quad \Delta \equiv (1, -2), \quad T \equiv (3, -1). \tag{12}$$

Their Yukawa couplings are

$$\delta\mathcal{L}_Y^{(1)} = c_{ij} (\bar{\nu}_{iL} \bar{e}_{iL}) \begin{pmatrix} e_{jR}^c \\ -\nu_{jR}^c \end{pmatrix} \bar{S} + d_{ij} (\bar{\nu}_{iL} \bar{e}_{iL}) \frac{\boldsymbol{\tau}\cdot \boldsymbol{T}}{\sqrt{2}} \begin{pmatrix} e_{jR}^c \\ -\nu_{jR}^c \end{pmatrix} + f_{ij} \bar{e}_{iL}^c e_{jR} \Delta^* + \text{h.c.}, \tag{13}$$

while, if we include also the right-handed neutrino, the following coupling is also possible

$$\delta\mathcal{L}_Y^{(2)} = h_{ij} \bar{N}_{iR} e_{jL}^c S + \text{h.c.}, \tag{14}$$

where $\bar{S} = (1, -1)$ could be the complex conjugate of S or a new field. Lepton number is conserved if we assign $L(S) = 2, L(T) = 2, L(\Delta) = 2$. Nevertheless, it would be unnatural to exclude cubic or quartic scalar boson couplings which can violate lepton number explicitly. The following lepton-number non-conserving couplings are possible:

$$\delta V = \mu H H' \bar{S} + \mu' H T H' + \mu'' S S \Delta + \mu''' T^2 \bar{\Delta} + \ldots + \text{h.c.} \tag{15}$$

The mass parameters $\mu, \mu'$ ... are arbitrary for the moment but it is natural to consider them of the same order of magnitude.

It should be pointed out that the Yukawa matrices $c, d, f$ in (13) and $h$ in (14), since they have nothing to do with the lepton mass matrices, are in general nondiagonal. The coupling of the singlet [6]

$$c_{ij}(\bar{\nu}_{iL} e^c_{jR} - \bar{e}_{iL}\nu^c_{jR})S^* + \text{h.c.} \tag{16}$$

is necessarily antisymmetric in family space due to SU(2) invariance, a feature quite interesting which leads to family mixing naturally. We should stress that this concludes all possibilities of lepton- and family-number violation in the framework of the standard gauge group. Guided by phenomenology and present experimental limits we shall obtain limits on the parameters of the new fields.

*Neutrino masses and mixings.* Left-handed neutrinos can acquire Majorana masses either through mixing with the right-handed neutrino or by radiative corrections due to the diagrams involving couplings of the new bosons [7]. The generated neutrino masses are:

$$\mathcal{L}^{(N)} \sim (L\cdot\bar{H})(bM^{-1}b)(H^\dagger\cdot L^c) + \text{h.c.}$$

$$= (\bar{\nu}_{iL})(\nu^c_{jR})(b_{ik}M_k^{-1}b_{kj})\langle\bar{H}_0\rangle^2 + \text{h.c.} \tag{17}$$

and

$$\mathcal{L}^{(S,T,\Delta)} \sim (L\cdot\bar{H})J_1(H^\dagger\cdot L^c) + (L\boldsymbol{\tau}J_2L^c)(H^\dagger\boldsymbol{\tau}\bar{H}) + \text{h.c.}, \tag{18}$$

where $J_1$ and $J_2$ are functions which are calculated from the loop integrals. The dominant contribution in (18) comes from the one-loop diagram of fig. 1 which gives

$$\mathcal{L}^{(S,T)} \sim (\bar{\nu}_{iL})(\nu^c_{jR})\left(\frac{c_{ij}}{16\pi^2}\mu\frac{m_j^2(e)-m_i^2(e)}{\mu_S^2-\mu_{H'}^2}\ln\frac{\mu_S^2}{\mu_{H'}^2} + \frac{d_{ij}}{16\pi^2}\mu'\frac{m_j^2(e)+m_i^2(e)}{\mu_T^2-\mu_{H'}^2}\ln\frac{\mu_T^2}{\mu_{H'}^2}\right). \tag{19}$$

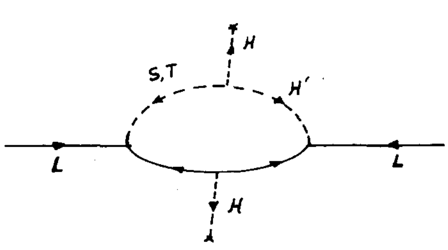


Fig. 1. Neutrino masses.

The presence of a second doublet is necessary and we have assumed that its couplings are diagonal and of the same order as the ones of the usual Higgs doublet. When the triplet is considered we must assume of course that no appreciable expectation value is associated with it so that the successful relation $M_Z = M_W \cos\theta_W$ is not spoiled.

The experimental limits on neutrino masses can readily translate into limits on the parameters appearing in (17) and (19). Since we are in no position to know something about the structure of $b$ and $M$, we only deduce from (17) that

$$(b^2\langle\bar{H}_0\rangle^2/M)_{\max} \lesssim O(10\ \text{eV}).$$

The natural value of $b\langle\bar{H}_0\rangle$ is of the order of the charged-lepton masses. Thus, for $b\langle\bar{H}_0\rangle \sim O(m_\tau)$, we get

$$M \gtrsim 10^8\ \text{GeV},$$

which is by the way in agreement with astrophysical limits in case $M$ is associated with the majoron [5]. The neutrino mass generated by (19) leads to analogous limits on the relevant parameters. In that case the neutrino mass matrix can be but in the form

$$(\mu/16\pi^2)(\mu_S^2-\mu_{H'}^2)\ln(\mu_S^2/\mu_{H'}^2)\begin{bmatrix} 0 & c_{e\mu}m_\mu^2 & c_{e\tau}m_\tau^2 \\ & 0 & c_{\tau\mu}m_\tau^2 \\ & & 0 \end{bmatrix}$$

$$+ (\mu'/16\pi^2)(\mu_T^2-\mu_{H'}^2)\ln(\mu_T^2/\mu_{H'}^2)$$

$$\times \begin{bmatrix} 2d_{ee}m_e^2 & d_{e\mu}m_\mu^2 & d_{e\tau}m_\tau^2 \\ & 2d_{\mu\mu}m_\mu^2 & d_{\mu\tau}m_\tau^2 \\ & & 2d_{\tau\tau}m_\tau^2 \end{bmatrix}. \tag{20}$$

Assuming $c_{e\mu} \simeq c_{e\tau} \simeq c_{\tau\mu} \equiv c$, $d_{e\mu} \simeq d_{e\tau} \simeq d_{\mu\tau} \equiv \bar{d}$ and $d_{ee} \approx d_{\mu\mu} \approx d_{\tau\tau} \equiv d$, we get

$$\mathcal{M}^{(S,T)}_{(\nu_L)} \begin{bmatrix} 2\delta m_e^2 & \gamma m_\mu^2 & \gamma m_\tau^2 \\ & 2\delta m_\mu^2 & \gamma m_\tau^2 \\ & & 2\delta m_\tau^2 \end{bmatrix}, \tag{21}$$

where

$$\delta \equiv (\mu'/16\pi^2)[d/(\mu_S^2-\mu_{H'}^2)]\ \ln(\mu_S^2/\mu_{H'}^2),$$

$$\gamma \equiv (\mu'/16\pi^2)[\bar{d}/(\mu_T^2-\mu_{H'}^2)]\ln(\mu_T^2/\mu_{H'}^2)$$
$$+ (\mu/16\pi^2)[c/(\mu_S^2-\mu_{H'}^2)]\ \ln(\mu_S^2/\mu_{H'}^2).$$

The eigenvalues of (21) can be estimated, assuming $O(\gamma) \approx O(\delta)$, to be

$$\mathcal{M}(\nu_1) \simeq (\delta - \gamma)m_\mu^2,$$

$$\mathcal{M}(\nu_2,\nu_3) \simeq \delta\,[1 \pm (1+2\gamma^2/\delta^2)^{1/2}]\,m_\tau^2. \qquad (22)$$

It is natural to assume that $\mu' \sim \mu \sim \mu_S \sim \mu_T$ and that $\mu_{H'} \sim O(m_W)$. Then

$$\mathcal{M}(\nu_1) = (\delta - \gamma)m_\mu^2\ ,$$
$$\sim O(\alpha/4\pi)[\mu m_\mu^2/(\mu^2-m_W^2)]\ \ln(\mu^2/m_W^2) \lesssim O(10\,\mathrm{eV}),$$

which translates into

$$\mu \gtrsim 10\ \mathrm{TeV}. \qquad (23)$$

An even stronger constraint can be obtained if one considers the induced neutrino oscillations [7]. In that case one gets $\mu \gtrsim 1000$ TeV.

It is highly unlikely of course that both neutrino mass mechanisms operate, if they operate at all. Although the right-handed neutrino mechanism is more economical it has the unpleasant feature that it involves very large mass scales and that it provides us with absolutely no information about the presence or not of family mixing. On the other hand, the boson-induced neutrino mass due to the presence of the antisymmetric couplings to S automatically involves big mixings independently of the actual value of the scale $\mu$.

*Muon decays.* It is well known that flavour mixing Higgs couplings lead to family-number violation in muon decays. The most popular such decay is $\mu^- \to e^-\gamma$ which can proceed via the diagram of fig. 2 provided that a flavour mixing coupling to a second Higgs doublet $L_i H' e_{jR}$ exists. We shall not devote any time to this well-studied case [‡1] but we proceed to analyze other possibilities.

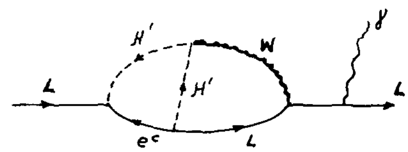


Fig. 2. $\mu \to e\gamma$ via flavour mixing Higgs couplings.

The $\Delta L = 2$ vertex $\bar{\nu}_L \nu_R^c$ induced by the right-handed neutrino contains off-diagonal elements but cannot lead to a $\mu_L^- e_L^{-*}$ operator unless it appears twice in which case it will be highly suppressed and therefore uninteresting. The new boson couplings on the other hand provide new possibilities. Among these possibilities the one realized by the singlet S seems most attractive since the existence of off-diagonal couplings is automatic.

*$\mu^- \to e^-\gamma$ via the singlet S.* Fig. 3 shows the relevant diagrams that involve an S, $\nu_L$ loop. The most general form for the on-shell ($q^2 = 0$) $\mu \to e\gamma$ amplitude is given by

$$\mathcal{M}(\mu \to e\gamma) = \bar{u}_e(F_M + F_M'\gamma_S)im_\mu\sigma_{\rho\nu}q^\rho\epsilon^\nu u_\mu, \qquad (24)$$

where $\epsilon^\nu$ is the photon polarization vector and $\sigma_{\rho\nu} = \frac{1}{2}i[\gamma_\rho, \gamma_\nu]$. Then, the decay rate is

$$\Gamma(\mu \to e\gamma) = (m_\mu^5/8\pi)(|F_M|^2 + |F_M'|^2). \qquad (25)$$

After the computation of $F_M, F_M'$ from the diagrams of fig. 3 the decay rate is found to be [‡2]

‡1 Since the late 1950's numerous papers have appeared. For a recent review see ref. [8].
‡2 For detailed calculations see ref. [9].

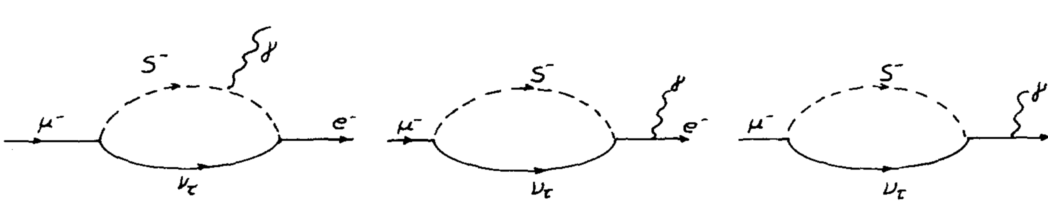


Fig. 3. $\mu \to e\gamma$ via the singlet S.

$$\Gamma(\mu \to e\gamma) = (e^2/4\pi)(c_{\mu\tau}c_{\tau e}/768\pi^2)^2\, m_\mu^5/\mu_S^4. \qquad (26)$$

On the other hand, the rate of the usual muon decay ($\mu^- \to e^-\nu_\mu\bar{\nu}_e$) is

$$\Gamma(\mu \to e\bar{\nu}_e\nu_\mu) = G_F^2 m_\mu^5/192\pi^3. \qquad (27)$$

Therefore the branching ratio will be

$$R_S = \Gamma(\mu \to e\gamma)/\Gamma(\mu \to e\bar{\nu}_e\nu_\mu)$$

$$= (\sin^4\theta_W/384\pi^3)(c_{\mu\tau}c_{\tau e}/e)^2(m_W/\mu_S)^4$$

$$\approx 4.48 \times 10^{-5}(c_{\mu\tau}c_{\tau e})^2(m_W/\mu_S)^4. \qquad (28)$$

For $c_{\mu\tau} \simeq c_{\tau e} \simeq 0.1$ we get

$$R_S \simeq 4.5 \times 10^{-9}(m_W/\mu_S)^4 \qquad (29)$$

which even for $\mu_S = 10$ TeV gives a rather suppressed ratio $R \leqslant O(10^{-17})$ very much below the present experimental limit $R_{exp} < 1.7 \times 10^{-10}$.

*$\mu^- \to e^-\gamma$ via the doubly charged singlet $\Delta$.* This case is entirely analogous to the previous one. The relevant diagrams are shown in fig. 4. We did not obtain a limit on $\mu_\Delta$ from the neutrino mass matrix since the related diagrams are at least of two-loop order and therefore this limit is covered by the lower limit on $\mu_S$. A limit on $\mu_\Delta$ can be obtained from $\mu^- \to e^+e^-e^-$ which can proceed at the tree level with one $\Delta$ exchange (fig. 6). As we shall see shortly we get $\mu_\Delta \gtrsim 3 \times 10^3$ GeV.

From the diagrams of fig. 4 with an explicit calculation we arrive at the result for the branching ratio

$$R_\Delta \simeq 2.5 \times 10^{-8}(m_W/\mu_\Delta)^4, \qquad (30)$$

where we have assumed that $f_{ee} \simeq f_{\mu e} \simeq 0.1$. Using the constraint obtained above for $\mu_\Delta$, we get

$$R_\Delta \approx 1.26 \times 10^{-14}. \qquad (31)$$

It is evident that $\mu^- \to e^-\gamma$ can also be mediated by the triplet T yielding a similar value for $R$.

We next turn to the $\mu \to 3e$ decay. This process is interesting from the experimental as well as from the theoretical point of view [8]. We will analyze the same possibilities as before.

*$\mu^\pm \to e^\pm e^+e^-$ via the singlet S.* There are three classes of one loop diagrams which are shown in fig. 5. We will start our discussion with the box diagram (fig. 5). The relevant amplitude takes the form

$$\mathcal{M} = (G_F/\sqrt{2})(k/\sqrt{2})(1 - P_{12})\bar{u}(p_2)\gamma_\nu(1+\gamma_5)v(p_e)$$

$$\times\, \bar{u}(p_1)\gamma^\nu(1+\gamma_5)u(p_\mu),$$

where

$$k = (1/32\pi^2)[(c_{e\mu}^2 + c_{\tau\mu}^2)c_{e\tau}c_{\mu\tau}/g^2]\, m_W^2 m_\mu^2/\mu_S^4, \qquad (32)$$

where $g$ is the usual SU(2) coupling constant. Therefore the branching ratio becomes

$$R = |k|^2. \qquad (33)$$

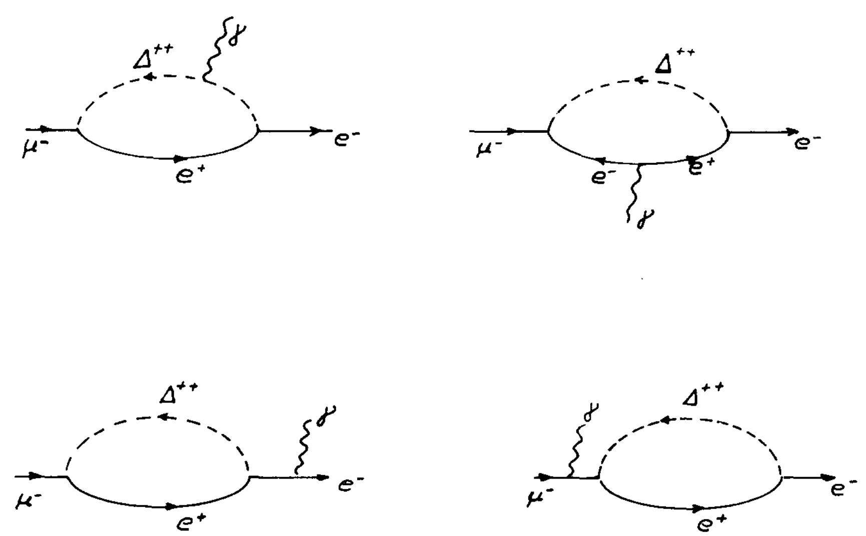


Fig. 4. $\mu \to e\gamma$ via the doubly charged singlet $\Delta$.

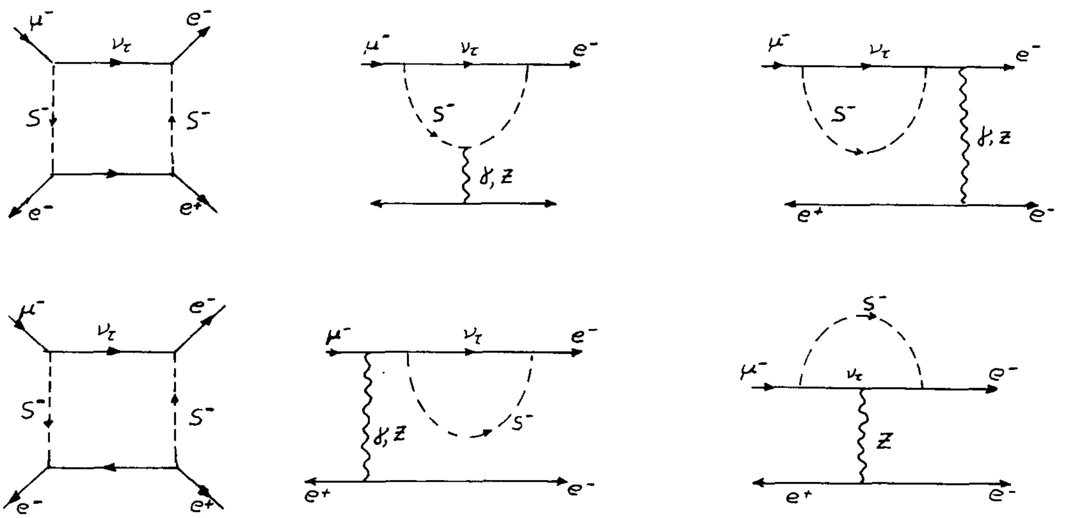


Fig. 5. $\mu \to 3e$ via the singlet S.

For $c_{e\mu} \approx c_{e\tau} \approx 0.1, \mu_S \sim 10^6$ GeV we get $R \approx 5 \times 10^{-30}$ which is much beyond the goals of present experiments. The branching ratio can be larger if the singlet S is substantially lighter but in any case about $10^7$ times smaller than the contribution of $\Delta$ and T (with similar masses and couplings).

There also exist diagrams mediated by the Z boson (fig. 5) but their contribution is negligible. The dominant contribution comes from those diagrams which involve a virtual photon. In the last case the decay rate is found to be

$$\Gamma(\mu \to 3e) \approx m_\mu^5 \tfrac{75}{24}(\alpha^2/\pi)|F_M|^2, \tag{34}$$

where $\alpha = e^2/4\pi$ and $F_M$ is the same as in relation (25). It is worthwhile to write the branching ratio

$$R_S = \Gamma(\mu \to 3e)/\Gamma(\mu \to e\gamma)$$
$$\approx \tfrac{75}{24}(\alpha/\pi) \approx 0.73 \times 10^{-2}, \tag{35}$$

which shows that $\mu \to e\gamma$ is preferable over $\mu \to 3e$ from the point of view of the rate, which has also been found to hold for conventional models [8].

*$\mu^\pm \to e^\pm e^+ e^-$ via the doubly charged singlet $\Delta$.* In this case we get from the tree level $\Delta$-exchange diagram of fig. 6

$$R_\Delta = \Gamma(\mu \to 3e)/\Gamma(\mu \to e\bar{\nu}_e \nu_\mu)$$
$$= \tfrac{1}{2}(f_{e\mu} f_{ee}/g^2)^2 (m_W/\mu_\Delta)^4. \tag{36}$$

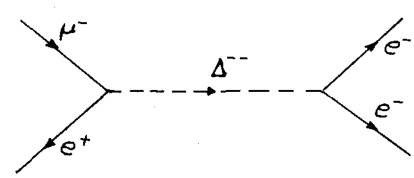


Fig. 6. $\mu \to 3e$ via the doubly charged singlet $\Delta$.

From the experimental limit $R \leqslant 1.6 \times 10^{-10}$ we obtain

$$(f_{ee} f_{e\mu}/\mu_\Delta^2) \lesssim 10^{-9}\ (\text{GeV})^{-2}. \tag{37}$$

For $f_{e\mu} \approx f_{ee} \approx 0.1$ we get the constraint

$$\mu_\Delta \gtrsim 3 \times 10^3\ \text{GeV}. \tag{38}$$

This is probably the most direct constraint on the doubly charged singlet. The process is observable in the future experiments even if $\mu_\Delta$ is an order of magnitude larger.

*Muonium–antimuonium ($m$–$\bar{m}$) oscillations.* Such m–$\bar{\text{m}}$ oscillations can in principle occur in the presence of interactions which can induce transitions of the type

$$\mu^+ \leftrightarrow e^+, \quad e^- \leftrightarrow \mu^-, \tag{39a}$$

$$\mu^+ \leftrightarrow \mu^-, \quad e^- \leftrightarrow e^+. \tag{39b}$$

They can occur, of course, via the mass mechanism discussed above if the neutrinos are massive and admixed. We notice, however that the mechanism (39a) occurs via the flavour changing ($\mu^-$, $e^-$) conversion twice while the other involves a mechanism analogous to that of neutrinoless double $\beta$-decay which also occurs twice. This implies that in the mass mechanism the oscillation period will be too long to be observed. This is not apparent in the discussion of Halprin [10] because the suppression due to the small mixing between the light and heavy neutrinos was not considered. The above processes can in principle be measurable if they are mediated by exotic Higgs particles [8]. The relevant amplitude takes the form

$$\mathcal{M} = (G_F/\sqrt{2}) K \bar{u}(p_2)(1+\gamma_5)v(p_1) \times \bar{v}(q_2)(1-\gamma_5)u(q_1), \tag{40}$$

where $p_1, p_2$ are the muon momenta and $q_1, q_2$ the electronic momenta. $K$ is a dimensionless quantity which will be given below for each model. Taking the non-relativistic limit of the amplitude (40) and using the 1S Coulomb wave functions for the electrons relative to the muon we get the oscillation

$$\gamma = (G_F/\sqrt{2})|K|\,|\varphi_{1S}(0)|^2 \simeq |K|(\alpha/\pi)G_F m_e^2, \tag{41}$$

which leads to an oscillation period

$$\tau = 1/\gamma = (1.3 \times 10^{-2}\ \mathrm{s})/|K|\,. \tag{42}$$

The value $K$ depends on the mechanism involved. Thus, when

(i) The process is mediated by the doubly charged isotriplet, then

$$K = 2(c_{\mu\mu}c_{ee}/g^2)(m_W/\mu_T)^2\,. \tag{43}$$

For $c_{ee} \approx c_{\mu\mu} \approx 0.1, \mu_T \sim 10^6$ GeV we get

$$\tau \geqslant 4 \times 10^7\ \mathrm{s} \approx 1.2\ \mathrm{yr}. \tag{44}$$

(ii) The process is mediated by the doubly charged singlet. Then eq. (40) remains the same except that $\gamma_5 \to -\gamma_5$ and

$$K = 2(f_{ee}f_{\mu\mu}/g^2)(m_W/\mu_\Delta)^2. \tag{45}$$

Using the bound obtained above on $\mu_\Delta, \mu_\Delta \gtrsim 3 \times 10^3$ GeV, we get

$$K \leqslant 3 \times 10^{-4}, \quad \tau \approx 400\ \mathrm{s}. \tag{46}$$

(iii) The process is mediated by S. In that case we get

$$K = (1/32\pi^2)[(|c_{e\tau}|^2 + |c_{\mu\tau}|^2)/g^2] \times (m_W/\mu_S)^2(m_\mu/\mu_S)^2. \tag{47}$$

For the above values of couplings we get

$$\tau \approx 2.3 \times 10^{26}\ \mathrm{s} \approx 5.1 \times 10^{19}\ \mathrm{yr}. \tag{48}$$

Two of us (G.K. Leontaris and J.D. Vergados) are happy to acknowledge partial support of this work by the Greek Ministry of Technology.

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